

5th Test On Advance Business Mathematics

FM: 50

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Duration: 2 hours 30 mins.

Group A

[2x5=10]

1. Answer the following questions:

a) If $f(x) = \frac{ax-b}{bx-a}$, show that $f(a).f\left(\frac{1}{a}\right) - f(b).f\left(\frac{1}{b}\right) = 0$

Or, If $f(x) = e^{ax+b}$, prove that $e^b f(x+y) = f(x).f(y)$

b) Find the domain of $\log(x^2 - 5x + 6)$

c) Find $\frac{dy}{dx}$, if $y = x^x$

Or, Find the stationary point of $f(x) = 2x^3 - 3x^2 - 12x + 4$

d) Find the value of x, y and z when $\begin{bmatrix} x+y & y-z \\ 1/2 & 7+x \end{bmatrix} = \begin{bmatrix} 4-x & z-4 \\ z-y & x+y+4 \end{bmatrix}$

e) If $P(A) = 2/3$, $P(B) = 1/2$, and $P(A \cup B) = 1$, then find the value of $P(A/B)$, $P(A/B^C)$ and $P(A^C \cap B^C)$.

Group B

[6x5=30]

2. Answer the following questions:

a) Solve by Cramer's Rule, the following system of linear equations:

$$x + 2y - z = -3, \quad 3x + y + z = 4, \quad x - y + 2z = 6$$

b) Evaluate (any two):

i. $\lim_{x \rightarrow 0} \frac{\sqrt{1+2x} - \sqrt{1-3x}}{x}$

iii. $\lim_{x \rightarrow 25} \frac{\sqrt{x}-5}{x-25}$

ii. $\lim_{x \rightarrow 2} \frac{x^2+x-6}{x^2-4}$

iv. $\lim_{x \rightarrow \infty} \frac{6x^3-3x^2+1}{2x^3+7x-4}$

c) Find $\frac{dy}{dx}$, if $x^y + y^x = 25$.

Or, If $y = \log(x + \sqrt{x^2 + a^2})$, prove that $(x^2 + a^2) \frac{d^2y}{dx^2} + x \frac{dy}{dx} = 0$

d) Find the maximum and minimum value of $x^{\frac{1}{x}}$, if there be any.

Or, The total cost of producing x units is Rs. C , where $C = \frac{x^3}{3000} - 0.2x^2 + 27x$ and all units produced are sold at Rs. 10 per unit. Find the output level such that marginal cost is equal to marginal revenue.

- e) Verify that the matrix equation $A^2 - 4A + 3I = O$ is satisfied by the matrix. Hence find A^{-1} , where $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$, $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $O = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Group C

3. Answer the following questions:

- a) Evaluate: (i) $\int \frac{x-1}{\sqrt{3x^2-6x+5}} dx$ (ii) $\int \frac{dx}{\sqrt{x+5}-\sqrt{x+3}}$ [2+2=4]
- b) (i) What is the chance that a leap year selected at random may have 53 Mondays?
 Find also the probability of having 53 Mondays in a non-leap year. [2]
- (ii) An executive committee of 6 members is to be formed from 4 ladies and 7 gentlemen. Find the probability that the committee will consist of (i) exactly two lady members (ii) at least 2 lady members. [4]